

AP Calculus

Mr. Liu

Name: _____

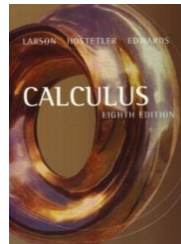
Textbook & Summer Work

Textbook

Calculus of a Single Variable, 8th Ed.

Larson, Hostetler, Edwards

Houghton Mifflin



Students are required to purchase their own textbook for this course. This should be done as soon as possible so that you can start preparing (see below). The textbook can be purchased from Amazon through one of their partner sellers. I've seen new copies selling for less than \$100, and used copies as low as \$25.

The purchase of the "Study and Solutions Guide" is optional. It provides the answers to all of the odd-numbered problems, as the textbook does, but also gives more explanation as to how the answers were derived. It can also be found on Amazon, new or used. Make sure to get the 8th edition, Volume 1 of this book.

Summer Work

In order to be successful in calculus, there are a number of skills that you have to be proficient with. Many of these skills were covered in your precalculus class, of course. But believe it or not, many calculus students get tripped up on algebra. The problems contained in this packet are intended to help you review these important topics.

Read through all of Chapter P in the textbook, which contains a review of algebra and precalculus. However, the problems in the packet go beyond just these topics. There are many resources that you can use for help. Here are some websites to check out.

www.purplemath.com

www.themathpage.com

www.analyzemath.com

www.mathwords.com

← Many terms and formulas

Do not work on this packet too soon. You don't want to finish everything by the end of June only to forget everything before September. I would suggest studying the relevant topics first, then working on the problems. These are only representative problems. The main goal is to understand the material, not just complete the packet.

Calculators will be an important learning tool in AP Calculus. All students are required to have a TI-84 calculator by the start of classes in the fall. Having said that, almost all of the problems in this packet should be done without a calculator. Get used to not using your calculator.

Answers are provided for selected problems at the end of the packet. As you have been told many times, the process is more important than the actual product. Therefore, concentrate on how to solve the problems, and show all work. Because you have time, solutions should be organized and written out neatly.

This packet is due on the first day of class and will be worth a quiz grade. They will not be accepted late. As you know, AP courses are designed to be college-level courses. Therefore, begin taking responsibility now!

Have a great summer, and see you in the fall. May the Lord bless you and your efforts.

Topic #1 – Fractional and Negative Exponents

Simplify using only positive exponents.

1. $2 \left(\frac{2}{2-x} \right) \left(\frac{-2}{(2-x)^2} \right)^{-3}$

2. $\frac{\sqrt{4x-16}}{\sqrt[4]{(x-4)^3}}$

3. $\frac{\frac{1}{2}(2x+5)^{-3/2}}{\frac{3}{2}}$

4. $\left(\frac{1}{x^{-2}} + \frac{4}{x^{-1}y^{-1}} + \frac{1}{y^{-2}} \right)^{-\frac{1}{2}}$

Topic #2 – Domain

Find the domain of the following functions.

1. $y = \frac{x^2-5x-6}{x^2-3x-18}$

2. $y = \frac{\sqrt{2x-9}}{2x+9}$

3. $y = \sqrt{x^2 - 5x - 14}$

4. $y = \log(2x - 12)$

Topic #3 – Solving Inequalities

Write the following absolute value function as a piecewise function.

1. $y = |2x - 4|$

Solve the following absolute value inequalities.

2. $|x - 3| \leq 4$

3. $|3x - 4| > -2$

Solve the following quadratic inequalities.

4. $x^2 - 3x \geq 10$

5. $x^3 + 4x^2 - x \geq 4$

6. $2 \sin^2 x \geq \sin x, 0 \leq x < 2\pi$

Solve the following rational inequality.

7. $\frac{2x-1}{3x-2} \leq 1$

Topic #4 – Even and Odd Functions

Algebraically determine if the relation is even, odd, or neither.

1. $f(x) = 2x^2 - 7$

2. $f(x) = -4x^3 - 2x$

3. $f(x) = 4x^2 - 4x + 4$

Topic #5 – Function Transformations

If $f(x) = x^2 - 1$, describe in words the following transformations.

1. $g(x) = x^2 - 3$
2. $g(x) = (x - 4)^2 - 1$
3. $g(x) = -(x + 2)^2 - 1$
4. $g(x) = 3x^2 - 7$
5. $g(x) = (2x)^2 - 1$
6. $g(x) = |x^2 - 1|$

Topic #6 – Rational Root Theorem (p/q)

Find all zeros of the function and factor completely.

1. $f(x) = x^3 - 6x^2 + 3x + 10$
2. $f(x) = x^3 + x^2 - 9x - 9$
3. $f(x) = 2x^3 - 3x^2 + 8x - 12$

Topic #7 – Special Factorization

Factor completely.

1. $27x^3 - 125y^3$
2. $x^4 + 11x^2 - 80$
3. $2x^2 - 20xy + 50y^2$
4. $x^2 + 12x + 36 - 9y^2$
5. $(x - 3)^2(2x + 1)^3 + (x - 3)^3(2x + 1)^2$
6. $\frac{1}{10}(2x + 1)^{5/2} - \frac{1}{6}(2x + 1)^{3/2}$
7. $(3x + 4)^{-3}(2x - 5)^3 + (3x + 4)^{-2}(2x - 5)^2$

Topic #8 – Solving Equations by Factoring or the Quadratic Formula

Solve each equation.

1. $x^2 + 6x + 4 = 0$
2. $x^3 + 2x^2 - 3x - 6 = 0$
3. $x - 10\sqrt{x} + 9 = 0$
4. $\frac{1}{x^2} - \frac{1}{x} = 6$
5. $2x^2 - (x + 2)(x - 3) = 12$

Topic #9 – Asymptotes

For each function, find the equations of both the vertical and horizontal asymptote(s), if they exist.

1. $y = \frac{x+4}{x^2-1}$
2. $y = \frac{x^2-9}{x^3+3x^2-18x}$
3. $y = \frac{2x^3}{x^3-1}$

Topic #10 – Complex Fractions

Simplify.

$$1. \frac{\left(\frac{3}{x} - \frac{4}{y}\right)}{\left(\frac{4}{x} - \frac{3}{y}\right)}$$

$$2. \frac{\left(1 - \frac{2}{3x}\right)}{\left(x - \frac{4}{9x}\right)}$$

$$3. \frac{\left(\frac{x^2 - y^2}{xy}\right)}{\left(\frac{x+y}{y}\right)}$$

$$4. \frac{x^{-3} - x}{x^{-2} - 1}$$

$$5. \frac{\left(\frac{4}{x-5} + \frac{2}{x+2}\right)}{\left(\frac{2x}{x^2 - 3x - 10} + 3\right)}$$

Topic #11 – Compositions of Functions

If $f(x) = x^2$, $g(x) = 2x - 1$, and $h(x) = 2^x$, find the following.

$$1. f(g(2))$$

$$2. h(f(-1))$$

$$3. g\left(f\left(h\left(\frac{1}{2}\right)\right)\right)$$

$$4. g(f(x))$$

$$5. g(g(x))$$

$$6. f(h(x))$$

Topic #12 – Rationalizing

Rationalize the denominator.

$$1. \frac{3x}{\sqrt{5}}$$

$$2. \frac{3}{\sqrt[4]{(3x)^3}}$$

Rationalize the numerator.

$$3. \frac{5\sqrt{2} + \sqrt{5}}{5}$$

$$4. \frac{\sqrt[3]{2x}}{7}$$

Topic #13 – Solving Rational Equations

Solve the equations for x .

$$1. \frac{x-5}{x+1} = \frac{3}{5}$$

$$2. \frac{60}{x} - \frac{60}{x-5} = \frac{2}{x}$$

$$3. \frac{2}{x+5} + \frac{1}{x-5} = \frac{16}{x^2-25}$$

$$4. \frac{x}{x-2} + \frac{2x}{4-x^2} = \frac{5}{x+2}$$

$$5. \frac{2x+3}{x-1} = \frac{10}{x^2-1} + \frac{2x-3}{x+1}$$

Topic #14 – Right Triangle Trigonometry

1. If $\cos \theta = -\frac{5}{13}$ and θ is in quadrant II, find $\sin \theta$ and $\tan \theta$.

2. If $\cot \theta = 3$ and θ is in quadrant III, find $\sin \theta$ and $\cos \theta$.

3. A kite is 100 m above the ground. If there are 200m of string out, what is the angle (in radians) between the string and the ground? (Assume the string is perfectly straight.)

Topic #15 – Solving Trigonometric Equations

Solve each equation on the interval $[0, 2\pi)$. Use a calculator for #5.

1. $\cos^2 x = \cos x$
 2. $4 \sin^2 x = 1$
 3. $2 \sin^2 x + \sin x = 1$
 4. $2 \sin x \cos x + \sin x = 0$
 5. $8 \cos^2 x - 2 \cos x = 1$
 6. $\sin^2 x - \cos^2 x = 0$
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Topic #16 – Logarithms

Simplify.

1. $\log_2 5 + \log_2(x^2 - 1) - \log_2(x - 1)$
 2. $2 \log_4 9 - \log_2 3$
 3. $3^{2 \log_3 5}$
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Topic #17 – Solving Literal Equations

Solve each equation for the indicated variable.

1. $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$, for a
 2. $A = 2\pi r^2 + 2\pi r h$, for $r > 0$
 3. $\frac{2x}{4\pi} + \frac{1-x}{2} = 0$ for x
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Topic #18 – Equations of Lines

Determine the equation of the line:

1. through $(-1, 3)$ and $(2, -4)$
 2. through $(-1, 2)$ and perpendicular to the line $2x - 3y + 5 = 0$
 3. through $(2, 3)$ and the midpoint of the line segment from $(-1, 4)$ to $(3, 2)$
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Topic #19 – Equations of Circles

For #1 and #2, given the circle $x^2 + y^2 + 6x - 4y + 3 = 0$, find:

1. the center and the radius
2. the equation of the tangent at $(-2, 5)$
3. A curve is traced by a point $P(x, y)$ which moves such that its distance from the point $A(-1, 1)$ is three times its distance from the point $B(2, -1)$. Determine the equation of the curve.

Topic #20 – The Difference Quotient

Simplify $\frac{f(x+h)-f(x)}{h}$, where:

1. $f(x) = 2x + 3$

2. $f(x) = \frac{1}{x+1}$

3. $f(x) = x^2$

Topic #21 – Inverse Functions

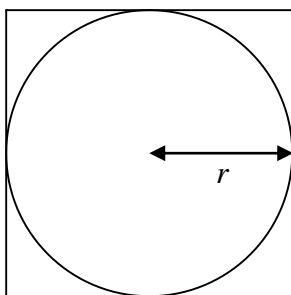
Find the inverse of each function.

1. $f(x) = 2x + 3$

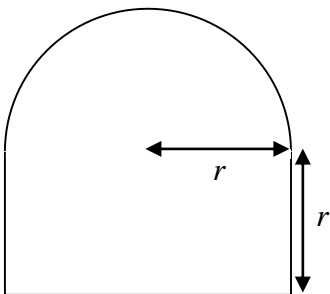
2. $f(x) = \frac{x+2}{5x-1}$

Topic #22 – Area

1. Find the ratio of the area inside the square but outside the circle to the area of the square.



2. Find a formula in terms of r for the perimeter of a window in the shape of a semicircle on top of a rectangle.



3. A water tank has the shape of a cone (like an ice cream cone) that is 10 m high and has a radius of 3 m at the top. The tank is partially filled with water. If the water is 7 m deep in the middle, what is the surface area of the top of the water?